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Temperature and Energy of 4-Dimensional Axisymmetric Black Holes from Entropic Force

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Abstract We investigate the temperature and energy on holographic screens for 4-dimensional axisymmetric black holes with the entropic force idea proposed by Verlinde. According to the principle of thermal equilibrium, the location of holographic screen outside the axisymmetric black hole horizon is not a equivalent radius surface. The location of isothermal holographic screen outside the axisymmetric black hole horizon is obtained. Using the equipartition rule, we derive the correction expression of energy of isothermal holographic screen. When holographic screens are far away the black hole horizon, the entropic force of charged rotating particles can be expressed as Newton's law of gravity. When the screen crosses the event horizon, the temperature of the screen agrees with the Hawking temperature and the entropic force gives rise to the surface gravity for both of the black holes.

Keywords Entropic force · Holographic screens · Axisymmetric spacetime · Thermal equilibrium

1 Introduction

Recently, Verlinde presented a remarkable new idea that gravity can be explained as an entropic force caused by the information changes a material body moves away from the holographic screens [1]. After his work, the dynamics of apparent horizon in the Friedmann-

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Robertson-Walker Universe [2, 3], the Friedmann equations [3–5], $f(R)$ gravity [6], deformed Horava-Lifshitz gravity [7], and braneworld scenario [8] were derived with the help of holographic principle and the equipartition rule of energy. The connection in the loop quantum gravity [9], the accelerating surfaces [10] holographic actions for black hole [11], and the application to holographic dark energy were considered from an entropic force [12]. An entropic force was discussed in the black hole spacetime [13–20].

According to Verlinde's idea, the holographic screens locate at equipotential surfaces, where the potential is defined by timelike Killing vector. Then the local temperature on a screen can be defined by the acceleration of a particle that is located very close to the screen. The energy on the screen is calculated by the holographic principle and the equitation rule of energy $E = \frac{1}{2}\int T dN$ with dN the bit density of information on the screen. References [14, 15], with the holographic principle and the equipartition law of energy, investigate the Unruh-Verlinde temperature and energy on holographic screens for several 4-dimensional black holes with static spherically symmetric metric and stationary axisymmetric metric. According to Ref. [14], outside the axisymmetric black hole horizon the Unruh-Verlinde temperature of every point on equivalent radius r holographic screens is difference. Therefore, when we take holographic screens as a thermodynamic system, holographic screens are unstable. In order to discuss the entropic force of an axisymmetric black hole horizon, we need determine the location of isothermal holographic screens outside the black hole horizon, and then investigate the physical properties of holographic screens outside the black hole horizon.

The paper is organized as follows. In Sect. 2, we review Verlinde's idea about the temperature and the energy from an entropic force in general relativity, and derive the Unruh-Verlinde temperature and energy on equivalent radius r holographic screens outside the axisymmetric black hole horizon. In Sect. 3, we discuss the Unruh-Verlinde temperature and energy of isothermal holographic screens outside the axisymmetric black hole horizon. We end this paper with conclusion in Sect. 4.

2 Entropic Force in the Presence of Axisymmetric Black Hole

In terms of Boyer-Lindquist coordinates, the Euclidean Kerr-Newman metric reads

$$ds^2 = -\frac{1}{\rho^2}\left(\Delta - a^2\sin^2\theta\right)dt^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 \\ + \frac{1}{\rho^2}\left[(r^2+a^2)^2 - \Delta a^2\sin^2\theta\right]\sin^2\theta d\varphi^2 - \frac{2a}{\rho^2}\left[(r^2+a^2) - \Delta\right]\sin^2\theta dt d\varphi, \quad (1)$$

where

$$\rho^2 = r^2 + a^2\cos^2\theta, \quad a = \frac{J}{M},$$

$$\Delta = r^2 + a^2 - 2Mr + q^2 = (r - r_+)(r - r_-), \quad (2)$$

$$r_\pm = M \pm \sqrt{M^2 - q^2 - a^2}.$$

M, J, q and $r_{+(-)}$ are mass, angular momentum, charge and outer and inner horizon locations of K-N black hole respectively, in the natural units of $\hbar = c = k_B = G = 1$.

From (1), we derive the metric determinant

$$g = -\rho^4 \sin^2\theta, \tag{3}$$

and the metric contravariant form

$$\begin{aligned} g^{00} &= -\frac{1}{\rho^2}\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right], \quad g^{11} = \frac{\Delta}{\rho^2}, \quad g^{22} = \frac{1}{\rho^2}, \\ g^{33} &= \frac{1}{\rho^2}\left(\frac{1}{\sin^2\theta} - \frac{a^2}{\Delta}\right), \quad g^{03} = -\frac{a(2Mr - Q^2)}{\Delta\rho^2}. \end{aligned} \tag{4}$$

In the general relativistic context one starts from a universal form of the Newtonian potential [14, 17, 21]

$$\phi = \frac{1}{2}\ln(-g^{\mu\nu}\xi_\mu\xi_\nu), \tag{5}$$

where ξ_μ satisfies the Killing equation

$$\nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu = 0. \tag{6}$$

The redshift factor is denoted by e^ϕ and it relates the local time coordinate to that at a reference point with $\phi = 0$, which we will take to be at infinity.

The acceleration is defined by the formula

$$a^\mu = -g^{\mu\nu}\nabla_\nu\phi, \tag{7}$$

and the Unruh-Verlinde temperature on the screen is given by the formula

$$T = -\frac{\hbar}{2\pi}e^\phi n^\mu a_\mu = \frac{\hbar}{2\pi}e^\phi n^\mu\nabla_\mu\phi = \frac{\hbar}{2\pi}e^\phi\sqrt{\nabla^\mu\phi\nabla_\mu\phi}, \tag{8}$$

where n_μ is a unit vector, that is normal to the holographic screen and the Killing time-like vector ξ_μ.

The corresponding potential is

$$\phi = -\frac{1}{2}\ln\left(\frac{(r^2+a^2)^2 - a^2\Delta\sin^2\theta}{\Delta\rho^2}\right), \tag{9}$$

which shows that equipotential surfaces are dependent with two parameters r and θ, hence the holographic screen is axisymmetric.

According to (7) and (9), we have

$$a^r = \frac{\Delta[(3r^2+a^2)M - rq^2] + (2Mr - q^2)(r^2+a^2)(M - r - \frac{\Delta r}{\rho^2})}{\rho^2[\Delta\rho^2 + (r^2+a^2)(2Mr - q^2)]}, \tag{10}$$

$$a^\theta = \frac{(2Mr - q^2)(r^2+a^2)a^2\sin(2\theta)}{2\rho^4[\Delta\rho^2 + (r^2+a^2)(2Mr - q^2)]}. \tag{11}$$

The Unruh-Verlinde temperature is given by

$$T = \frac{\hbar}{2\pi}\left\{\frac{[\Delta\rho^2((3r^2+a^2)M - rq^2) + (2Mr-q^2)(r^2+a^2)((M-r)\rho^2 - \Delta r)]^2}{\rho^4[\Delta\rho^2 + (r^2+a^2)(2Mr-q^2)]^3} + \frac{\Delta[(2Mr-q^2)(r^2+a^2)a^2\sin(2\theta)]^2}{4\rho^4[\Delta\rho^2+(r^2+a^2)(2Mr-q^2)]^3}\right\}^{1/2}. \tag{12}$$

The entropy force on a particle located very close to the holographic screen can be calculated as [21]

$$F^\mu = -me^\phi \nabla^\mu \phi, \tag{13}$$

where m is the mass of the particle. Obviously, with the help of (7), $F^\mu = me^\phi a^\mu$ is Newton's second law where e^ϕ is due to the redshift. The magnitude of this force is

$$F = \sqrt{F^\mu F_\mu} = 2\pi\hbar^{-1} mT. \tag{14}$$

When the screen coincides with the black hole horizon, we have

$$F_{r=r_\pm} = m\frac{r_+ - r_-}{2(r_\pm^2 + a^2)} \approx \frac{mM}{r_\pm^2}\left[1 - \frac{a^2}{r_\pm^2} - \frac{a^2+q^2}{2M^2} + \frac{a^2(a^2+q^2)}{2M^2 r_\pm^2}\right]. \tag{15}$$

Since we are interested in the behavior near the black hole horizon, we make a Taylor series expansion for (12) outside the horizon

$$T(r) = \frac{\hbar}{4\pi}\frac{r_+ - r_-}{r_+^2 + a^2} + T'(r_+)(r - r_+), \tag{16}$$

where

$$T'(r_+) = -\frac{\hbar}{2\pi}\left[\frac{r_-^2 + a^2}{(r_+^2+a^2)^2} + \frac{3r_+(r_+ - r_-)}{\rho^2(r_+^2+a^2)} + \frac{3(r_+ - r_-)r_+}{2(r_+^2+a^2)^2} + \frac{3(r_+ - r_-)^2\rho^2}{4(r_+^2+a^2)^3}\right] + \frac{\hbar}{8\pi}\frac{[a^2\sin(2\theta)]^2}{\rho^4(r_+^2+a^2)}. \tag{17}$$

From (14), near the black hole horizon, the magnitude of this force is

$$F(r) = 2\pi\hbar^{-1} mT(r), \tag{18}$$

where $T(r)$ is given by (16). According to (18), (16) and (17), the values of $F(r)$ are not the same near the black hole horizon. The value is related to polar angle θ. The Unruh-Verlinde temperature is different on equal radius r holographic screens outside the black hole horizon. It is shown that for axisymmetric spacetime, the Unruh-Verlinde temperature of every point on equal radius r holographic screens outside the black hole horizon is different. Thus holographic screens are not isothermal. If we take the equal radius r holographic screens as a thermodynamic system, this system is unstable. In order to make holographic screens stable, we need find isothermal holographic screens outside the black hole horizon. Only on the stable holographic screens, discussing the magnitude of force is physically significant.

3 Isothermal Holographic Screens

In this section, we will find the isothermal holographic screens outside the black hole horizon. $T(r)$ given by (16) is a approximate expression on equivalent r surface outside the black hole horizon. In (16), let

$$r = r_+ + \frac{\delta}{f(\theta)}, \tag{19}$$

where δ is a positive small quantity, which is the same dimension as r. Since $f(\theta)$ is a dimensionless coefficient, according to (19), (17) and (16), when

$$f(\theta) = \left[\frac{r_-^2 + a^2}{(r_+^2 + a^2)} + \frac{3r_+(r_+ - r_-)}{\rho^2} + \frac{3(r_+ - r_-)r_+}{2(r_+^2 + a^2)} + \frac{3(r_+ - r_-)^2\rho^2}{4(r_+^2 + a^2)^2}\right] - \frac{1}{4\pi}\frac{[a^2\sin(2\theta)]^2}{\rho^4}, \tag{20}$$

we have

$$T(\delta) = \frac{\hbar}{4\pi}\frac{r_+ - r_-}{r_+^2 + a^2} - \frac{\hbar}{2\pi(r_+^2 + a^2)}\delta. \tag{21}$$

So near the black hole horizon, on isothermal holographic screens $r = r_+ + \frac{\delta}{f(\theta)}$, the magnitude of force is not related to polar angle θ, and

$$F(\delta) = 2\pi\hbar^{-1}T(\delta). \tag{22}$$

Now we apply the equipartition relation to calculate the energy on the holographic screen. Assuming the total number of bits N on the holographic screen being proportional to the area of the screen A, $N = A$, we have

$$E = \frac{1}{2}\int_s T dN = \frac{1}{2}\int_s T dA = \frac{1}{2}\int_s T\sqrt{(r^2 + a^2)\rho^2 + (2Mr - q^2)a^2\sin^2\theta}\,\sin\theta d\theta d\varphi, \tag{23}$$

where

$$dA = \sqrt{(r^2 + a^2)\rho^2 + (2Mr - q^2)a^2\sin^2\theta}\,\sin\theta d\theta d\varphi.$$

We obtain the energy on the horizon

$$E_{r=r_+} = \sqrt{M^2 - a^2 - q^2}. \tag{24}$$

The area of isothermal holographic screens $r = r_+ + \frac{\delta}{f(\theta)}$ is

$$A \approx 4\pi(r_+^2 + a^2) + \Delta A, \tag{25}$$

where

$$\Delta A \approx \int\left[\frac{r_+(2r_+^2 + a^2 + a^2\cos^2\theta) + Ma^2\sin^2\theta}{(r_+^2 + a^2)}\frac{\delta}{f(\theta)}\right]\sin\theta d\theta d\varphi \tag{26}$$

is increment of isothermal holographic screens $r = r_+ + \frac{\delta}{f(\theta)}$ corresponding the black hole horizon surface. Substituting (21) and (25) into (23), we can obtain the energy of constant acceleration screen outside the rotating black hole horizon.

$$E \approx E_{r=r_+} - \delta + \frac{(r_+ - r_-)}{8\pi(r_+^2 + a^2)}\Delta A. \tag{27}$$

Because isothermal holographic screens $r = r_+ + \frac{\delta}{f(\theta)}$ are near the horizon, ΔA is a positive small quantity. This term is a small correction term to the energy of constant acceleration screen.

When $r \gg r_+$,

$$T(r) \approx \frac{1}{2\pi} \frac{r^4(Mr^2 - q^2 r + 2Ma^2 + 3Mq^2)}{\sqrt{[r^4 + a^2 r(r + 2M)]^3}}, \tag{28}$$

i.e. isothermal holographic screens are spherical surface. Thus

$$\begin{aligned} E &= \frac{1}{2}\int_s T dN = \frac{1}{2}\int_s T dA = \frac{1}{4\pi}\int_s \frac{r^4(Mr^2 - q^2 r + 2Ma^2 + 3Mq^2)}{\sqrt{[r^4 + a^2 r(r + 2M)]^3}} dA \\ &\approx \left(\frac{Mr^2 - q^2 r + 2Ma^2 + 3Mq^2}{r^2}\right)\left(1 - \frac{a^2}{2r^3}(r + 2M)\right)^3 \\ &\approx M\left(1 + \frac{1}{2}\left(\frac{a}{r}\right)^2\right) - \frac{q^2}{r} + \frac{3Mq^2}{r^2}. \end{aligned} \tag{29}$$

When $a \to 0$, spacetime line element (1) degenerates to Reissner-Nordstrom spacetime. Our result (15) returns to that of Ref. [14]. From (15), when $r \to \infty$, in Kerr-Newman black hole the effect of angular momentum a and charge q per unit mass on the energy can be neglected. When $r \to \infty$, and the high order terms have been neglected, the effect of Kerr-Newman black hole on the energy of acceleration screen is the same as that of Schwarzchild black hole.

4 Conclusion

In this paper, with the holographic principle and the equipartition law of energy, we investigate the Unruh-Verlinde temperature and energy on screens for 4-dimensional black hole with stationary axisymmetric metric. Because the temperature on holographic screens outside the axisymmetric black hole horizon vicinity is different, holographic screens is thermodynamic unstable. On the thermodynamic unstable holographic screens, The Unruh-Verlinde temperature and energy are unreliable. Finding isothermal holographic screens outside the axisymmetric black hole horizon and investigating the Unruh-Verlinde temperature and energy on isothermal holographic screens are physical meaningful. It makes people have further comprehension on the entropic force that exist about the axisymmetric black hole horizon.

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